Impulse & Momentum

Week 7, Lesson 2

- **Linear Momentum**
- **Impulse**
- **Conservation of Linear Momentum**
- **Collisions**
- **Centre of Mass**

References/Reading Preparation:
Schaum's Outline Ch. 8
Principles of Physics by Beuche – Ch.6

The Concept of Linear Momentum

An experience common to us all is that a moving object possesses a property that causes it to exert a force on anything or anyone trying to stop it.

- The faster an object is moving, the harder it is to stop.

and

- The more massive the object is, the more difficulty we have in stopping it.

For example, an automobile moving at 2 m/s is more difficult to stop than a bicycle moving at 2 m/s.

We call this the Linear Momentum of the moving object.

The **Linear Momentum** of a body is the product of its mass and velocity.

Linear Momentum = **p** = mv

Momentum is a vector quantity whose direction is that of the velocity.

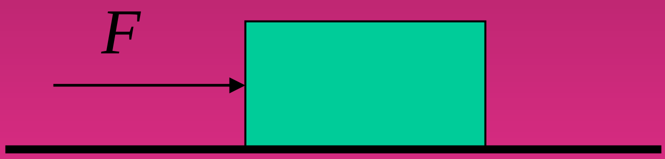
Its units are N·s in the SI.

Newton's Second Law Restated - Impulse

There is an important relationship between the net force applied to an object and the change in momentum that the force causes.

When a net force F accelerates an object, it obviously causes the velocity , and thus the momentum, of the object to change.

Consider an object of mass m as shown.



Because of the force applied, the object experiences an acceleration.

Now, $F = ma$, and $a = (v_f - v_o)/t$

Substituting for a, we get: $F = \dfrac{m(v_f - v_o)}{t} = \dfrac{mv_f - mv_o}{t}$

$$= \frac{\Delta p}{t}$$

This equation says that the net force that acts on an object equals the time rate of change of the object's linear momentum.

Rewritten, the equation is:

$$Ft = mv_f - mv_o$$

where:

Ft = the **impulse** (the product of a force and the time the force acts.)

$mv_f - mv_o$ = the change in momentum

An impulse causes a change in momentum.

Conservation of Momentum

If the resultant external force on a system of objects is zero, then the vector sum of the momenta of the objects will remain constant.

In collisions:

The vector sum of the momenta just before the event equals the vector sum of the momenta just after the event.

The vector sum of the momenta of the objects involved does not change during the collision.

Thus, when two bodies of masses m_1 and m_2 collide,

Total momentum before impact = total momentum after impact

or,

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

where,

$\mathbf{u}_1$ and $\mathbf{u}_2$ are the velocities before impact

$\mathbf{v}_1$ and $\mathbf{v}_2$ are the velocities after impact

In component form:

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

and

$$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$$

An **inelastic collision** is one during which kinetic energy is lost.

These are collisions where two bodies stick together after the collision.

A **perfectly elastic collision** is one during which KE is conserved.

These are collision where the bodies 'bounce' off each other.

For these types of collisions;

Total KE before the collision = total KE after the collision

or

$$\tfrac{1}{2} m_1 u_1^2 + \tfrac{1}{2} m_2 u_2^2 = \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2$$

Types of Momentum Questions:

- bullet fired into a block- final velocity given
- bullet fired into a block – final height given
- elastic collisions where one bounces off the other – 1-D and 2-D
- inelastic collisions – where they stick together after the collision
- impulse questions – bat and ball, etc
- centre of mass

Centre of Mass

The centre of mass of an object is the single point that moves in the same way as a point mass would move when subjected to the same external forces that act on the object.

If the resultant force acting on an object (or system of objects) of mass m is F, the acceleration of the centre of mass of the object (or system) is given by $\mathbf{a}_{cm} = \mathbf{F}/m$.

Centre of Mass (cont'd)

If the object is considered to be composed of tiny masses m_1, m_2, m_3, and so on, at coordinates (x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3), etc., then the coordinates of the centre of mass are given by:

$$x_{cm} = \frac{\Sigma x_i m_i}{\Sigma m_i} \qquad y_{cm} = \frac{\Sigma y_i m_i}{\Sigma m_i} \qquad z_{cm} = \frac{\Sigma z_i m_i}{\Sigma m_i}$$

Where the sums extend over all the masses composing the object.

In a uniform gravitational field, the centre of mass and the centre of gravity coincide.

Centre of mass examples:

Three masses are placed on the x-axis: 200g at x = 0, 500 g at x = 30 cm, and 400 g at x = 70 cm. Find their centre of mass.

(ans. 0.391 m)

Centre of mass example

A system consists of the following masses in the xy-plane: 4 kg at (x = 0, y = 5 m), 7 kg at (3 m, 8 m), and 5 kg at (-3 m, -6 m). Find the position of its centre of mass.

(ans. 0.375, 2.875)

Example:

An 8 g bullet is fired horizontally into a 9 kg block of wood and sticks in it. The block, which is free to move, has a velocity of 40 cm/s after impact. Find the initial velocity of the bullet.

(ans. 450 m/s)

Example:

A 16 g mass is moving in the +ve x-direction at 30 cm/s while a 4g mass is moving in the –ve y-direction at 50 cm/s. They collide and stick together. Find their velocity after the collision.

(0.140 m/s)

Impulse example

A o.35 kg ball moving in the +x-direction at 13 m/s is hit by a bat. Its final velocity is 19 m/s in the –x-direction. The bat acts on the ball for 0.010 s. Find the average force F exerted on the ball by the bat.

(ans. –800N)